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Teachers in Industry for Educational Support

Lessons in Uncertainty Analysis

Developed by:
Chris Fisher, Miamisburg HS
Mathematics

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Lessons in Uncertainty Analysis
Curriculum Unit Overview

Summary

When measuring quantities, a researcher must ask, “How reliable are these results?” Error is defined to be the difference between the measured value and the true value. How do we know what the true value is? An uncertainty value is the researcher’s best estimate of the error. It is an attempt to quantify how accurate results are believed to be. Engineers use advanced statistics and calculus to make these estimations. The intent of this unit is to simply introduce high school students to uncertainty analysis.

This unit can be adapted for students ranging in ability from typical 8^{th} grade to Calculus. The more advanced students will move through more quickly, and there will be more opportunity to discuss the topics more in depth.

Preparation for the Unit

Access to spreadsheet software, such as Excel, is necessary to conduct the technology portion of this unit. The science and mathematics portions can be done without it.

Overview

Science	Math	Technology
ALT 1 –Water Density Data to calculate water (and oil) density will be collected and an uncertainty will be determined.	**ALT 1 – Water Density** Data to calculate water (and oil) density will be collected and an uncertainty will be determined.	**ALT 1 – Water Density** A spreadsheet will be used to analyze the data.
ALT 2 – Moving Objects Data to calculate the rate of moving objects will be collected and an uncertainty will be determined.	**ALT 2 – Moving Objects** Data to calculate the rate of moving objects will be collected and an uncertainty will be determined.	**ALT 2 – Moving Objects** A spreadsheet will be used to analyze the data.
ALT 3 – Flow Rate Data to calculate flow rate will be collected and an uncertainty will be determined.	**ALT 3 – Flow Rate** Data to calculate flow rate will be collected and an uncertainty will be determined.	**ALT 3 – Flow Rate** A spreadsheet will be used to analyze the data.
Transfer Activity – Falling Objects Students will collect data to determine the equation that gives distance as a function of time for falling objects with no initial velocity. An uncertainty analysis will be done to quantify the accuracy of their results. The previous activities will help students explore ways of collecting data so that the uncertainty is minimized.		

ALT Hook: Uncertainty Analysis

Summary

Students will be exploring the basic notions of error and uncertainty.

Competencies

At the end of this lesson students will be able to describe the basic notions of uncertainty analysis

Time

This lesson may take up to a 30 minutes

Materials

1. Stop watches
2. Rulers
3. Squares
4. Two large beaker of water

Instructions

1. Give students each a stop watch and ask, “How fast can you START and STOP a stopwatch?” Have a contest and record the results on the board.
2. Fill two large beakers with the same amount water and ask, “How much water is in each of these containers?” Add and subtract a few drops of water from one and repeat the question.
3. Give students squares, or have them draw their own and ask them to measure the diagonal as accurately as possible. “How long is the diagonal of this square?”

Evaluation/Assessment of Student’s Competency

Students will be assessed based on their participation

Closure

We will be conducting experiments in the next few lessons in order to explore the concept of uncertainties and the effect of uncertainty on research.

ALT One: Water Density

Summary

Students will be collecting data to calculate the density of water and oil. The primary focus will be on finding the uncertainty of the calculations, which is a quantity that measures the level of accuracy with which the data was collected.

Competencies

At the end of this lesson students will be able to:
1. describe some basic ideas about uncertainty analysis
2. identify where errors may occur in collecting data
3. identify ways to limit error in data collection
4. organize data in tables and graphs
5. compute measures of central tendency
6. collect data using scales and measuring devices

Time

This lesson may take up to 3 hours to complete.

Materials

1. Containers larger than one gallon
2. Measuring cups
3. Vegetable oil
4. Graph paper
5. Yard/Meter sticks and rulers
6. Scale(s)
7. Handouts
8. Calculators (optional)
9. Spreadsheet software

Instructions

1. Fill one container with water and one with oil to be used as the class samples. Everyone will weigh this container in order to determine the groups' accuracy in weighing.
2. Pair students together.
3. Read through the instructions with students. Data will be shared so everyone will have the same information recorded in the tables. As students bring up their samples, measure the liquid heights with them and record them on the board or overhead.
4. Discuss questions

5. Read through the calculation instructions with students, and have them fill out the tables in their groups and answer the follow up questions. (NOTE: We are assuming here that the averages are the “true” values. Hopefully this will not set well with some students.)
6. Discuss questions
7. Have students organize their organize their data in a spread sheet. Take some time to teach them how to program the spreadsheet to do calculations and make graphs representing the data.

Evaluation/Assessment of Student’s Competency

Each group will turn in a copy of their tables, graphs and questions to be graded.

Closure

This lesson gives students the basic notions of uncertainty analysis, so be sure to compare your results with the true density of water ~62.4lb/ft^3. End the final class discussion with this example and connect it to the data they collected on density. The intent of follow up question 1 is to get students to understand this.

Rate = D/T
Distance is 100 ft, plus or minus one foot (1%)
Time is 20 sec, plus or minus one second (5%)
Rate is 100/20 = 5 ft/sec…. Probably!
Rate is at most 101/19 = 5.3158…ft/sec
Rate is at least 99/21 = 4.7143…ft/sec
Uncertainty in Rate is **apporx 0.3 ft/sec** or 6%

Explain that the challenge is to determine how accurate the measurement is. (…plus or minus what?) The next lesson deals with DiRT.

Measurement of Water (and Oil)

<u>STEPS</u>:

1. Weigh the container filled with water and the empty container at the front of the room to calculate the weight of the water.

   W water = W full – W empty.

2. Use the measuring cup to fill the large container with one gallon of water. (How many cups will you need to measure out one gallon?)
3. Weigh **your** container and set it on the table in front.
4. Fill in the W water and Volume Height columns for each group in the tables below.

Group	W water (class sample)	W water (group samples)	Volume Height
Average			

5. Make a bar graph representing the data in your table.

<u>Questions</u>:

1. Why are there variations in the weight? What does this indicate about your individual water weight calculations?
2. Why are the heights of water in the containers slightly different?
3. What would happen if:
   a. We changed the size of our small container?
   b. We increased/decreased our final volume?
   c. The size or shape of our large container was different?

<u>Extension</u>: Repeat using vegetable oil.

Group	W oil (class sample)	W oil (group samples)	Volume Height
Average			

<u>Questions</u>:

1. Why are there variations in the weight? What does this indicate about your individual oil weight calculations?

2. Why are the heights of oil in the containers slightly different?

3. What would happen if:
    a. We changed the size of our small container?

    b. We increased/decreased our final volume?

    c. The size or shape of our large container was different?

Data for Density Calculations

<u>Instructions</u>:

1. Copy the W water (group samples) and Volume Height into the new tables
2. Fill out the Weight Uncertainty table
   a. Average the W water data
   b. Find the distance each W water measure is from the average
      Dist = W water - Average
   c. Find the range
      Range = High dist – Low dist
   d. Calculate the percentage
      % = (Dist / Average)100
   e. Find the range
      Range = High % – Low %
   f. Calculate Uncertainty
      Uncertainty = range / 2 ?
3. Fill out the Volume Uncertainty Table
   a. Average the Volume Height
   b. Find the distance each VH is from the average
      Dist = VH – Average
   c. Find the range
      Range = High dist – Low dist
   d. Calculate the percentage
      % = (Dist / Ave)100
   e. Find the range
      Range = High % – Low %
   f. Calculate the Amount of Error (in gallons)
      Error = Dist / Average
   g. Find the range
      Range = High Error – Low Error
   h. Calculate the Gallons measured
      Gallons = 1 + Error
   i. Calculate Uncertainty
      Uncertainty = range / 2 ?
4. Fill out the Density Uncertainty Table
   a. Copy the W water (group samples) and Volume (last column)
   b. Calculate each density
5. Fill in W water and Volume uncertainties… leave Density uncertainty blank.

Density and Uncertainty Calculations

Water Data

Weight Uncertainty for Water			
Group	W water (class sample)	Distance From Average	% From Average
	Average =	Range =	Range =
Uncertainty =		Uncertainty % =	

Volume Uncertainty for Water					
Group	Volume Height	Distance From Average	% From Average	Amount of Error (in gal)	Gallons Measured
	Average =	Range =	Range =	Range =	
Uncertainty =			Uncertainty % =		

Density Uncertainty for Water			
Group	W water (group samples)	Volume	Density
Uncertainties			

Density and Uncertainty Calculations

Oil Data

Weight Uncertainty for Oil			
Group	W oil	Distance From Average	% From Average
	Average =	Range =	Range =
Uncertainty =		Uncertainty % =	

Volume Uncertainty for Oil					
Group	Volume Height	Distance From Average	% From Average	Amount of Error	Gallons Measured
	Average =	Range =	Range =	Range =	
Uncertainty =			Uncertainty % =		

Density Uncertainty for Oil			
Group	W oil	Volume	Density
Uncertainties			

Follow up questions

1. Use our uncertainties, for weight and volume, to find the largest and smallest **possible** densities we **could have** calculated. Explain.

2. Calculate the uncertainty of our density. Explain your procedure.

3. How confident are you that our calculated density is accurate? Explain.

4. What could we have done differently to improve our uncertainty? Explain.

<u>Extension</u>: Compare the densities of water and oil. How can we check to be sure that our results are correct? (Ever heard the expression, "Water and oil don't mix!"?)

ALT Two: Moving Objects

Summary

Students will be collecting data to calculate the rate of a toy car. The primary focus will be on finding the uncertainty of the calculations, which is a quantity that measures the level of accuracy with which the data was collected.

Competencies

At the end of this less students will be able to:
1. describe the concept of uncertainty analysis
2. quantify an uncertainty for a given set of data
3. identify where errors may occur in collecting data
4. identify ways to limit error in data collection
5. organize data in tables and graphs
6. compute measures of central tendency
7. collect data measuring time and distance

Time

This lesson may take up to 2 hours to complete.

Materials

1. Toy cars (or some other moving object)
2. Yard/meter sticks
3. Stop watches
4. Handouts
5. Graph paper
6. Calculators (optional)
7. Spreadsheet software

Instructions

1. Put students in groups of 2 to 4.
2. Mark off distances on the floor.
3. Determine an uncertainty for recording time and distance then answer questions 1 – 3.
4. Discuss questions.
5. Brainstorm with the class on what the rates of the cars are and how to best calculate the rate of the cars. (This is a good opportunity to talk about how the rates may change over time leading to discussions on changing slope and derivatives. “What is our instantaneous rate?”) Pass out the third handout and explain how the data will be collected. Ideas brought up during brainstorming may be helpful in seeing how to minimize the uncertainty (question 4).

6. Graph the rates over time. Construct a class graph of everyone's data. (If groups put their graph on transparencies, then lay them on one another to show the class graph.) You could also have them construct distance/time graphs. These may shed some light on the relations between distance, rate or velocity, and acceleration, which come up in the transfer activity. This is a great opportunity to introduce students to the basics of calculus.
7. Discuss questions and graphs. (If the rates of the cars are very different, it may be helpful to have the groups exchange cars and repeat the activity to compare differences in the rates.)
8. Have students organize their organize their data in a spread sheet. Assess their ability to program the spreadsheet to do calculations and make graphs representing the data. At this point the groups should be working more independently on the spreadsheet. Share a few of these with the entire class. (Consider, if possible, having the students program the spreadsheet prior to collecting data either in this lesson or the next lesson.)

Evaluation/Assessment of Student's Competency

Each group will turn in a copy of their tables and questions to be graded. (Be sure to give feedback! At the end of the next lesson, groups will be asked to present their work. Your feedback here is important in preparing them to share ideas with the class.)

Closure

Reinforce the notion of how the uncertainties build on one another…go back to the example:

Rate = D/T
Distance is 100 ft, plus or minus one foot (1%)
Time is 20 sec, plus or minus one second (5%)
Rate is 100/20 = 5 ft/sec…. Probably!
Rate is at most 101/19 = 5.3158…ft/sec
Rate is at least 99/21 = 4.7143…ft/sec
Uncertainty in Rate is **apporx 0.3 ft/sec** or <u>6%</u>

Discuss weather or not this showed up in the example. Most likely, the rates will fall in a cluster in the middle, with few or none at the extreme. (At the end of the unit there is an opportunity to discuss some rather advanced ideas from calculus and statistics on how to narrow the uncertainty.)

Also discuss the rates of the cars. Most likely, students saw the rates slowing down. Ask how things would have looked, graphically and in the tables, if the rates had remained constant. The next lesson deals with flow rates that should remain rather constant. (What about when the rate is changing at a constant rate? …constant acceleration?)

Moving Objects (DiRT)
Measured Uncertainty

Half of us will time, and half of us will record the distance of a single trial (which we may repeat) in order to quantify our ability to accurately measure time and distance. When I let go of the car, time recorders will start their watches. When I yell “STOP”, time recorders will stop their watches and distance recorders will estimate the distance traveled.

<table>
<tr><th>Time (sec)</th><th colspan="2">Dist from Ave</th><th>% form Ave</th></tr>
<tr><td></td><td colspan="2"></td><td></td></tr>
<tr><td>Ave =</td><td colspan="2">Range =</td><td>Range =</td></tr>
<tr><td colspan="2">Uncertainty =</td><td colspan="2">Uncertainty % =</td></tr>
</table>

<table>
<tr><th>Distance (ft)</th><th colspan="2">Dist. from Ave</th><th>% from Ave</th></tr>
<tr><td></td><td colspan="2"></td><td></td></tr>
<tr><td>Ave =</td><td colspan="2">Range =</td><td>Range =</td></tr>
<tr><td colspan="2">Uncertainty =</td><td colspan="2">Uncertainty % =</td></tr>
</table>

Moving Objects (DiRT)
Calculated Uncertainty

1. Use the average distance and time to determine the **rate** of the car.

2. Determine the largest and smallest possible rates we could have calculated from all of the previously recorded times and distances.

3. What is the uncertainty of the rate? Explain.

Moving Objects (DiRT)
Verifying Rate

1. Complete the following table.

Distance (ft)	Time (sec)	Rate (ft/sec)	Average Rate
1			
1			
1			
2			
2			
2			
3			
3			
3			
4			
4			
4			
5			
5			
5			
6			
6			
6			
7			
7			
7			

2. Does your average rate fall into the range from question 2 on the Calculated Uncertainty page?

3. How confident are you in the accuracy of your final average rate? (What is the uncertainty of your rate calculation?) Explain.

4. Are there any factors other than our ability to accurately measure time and distance that could have contributed to the variability in rates calculated?

ALT Three: Flow Rate

Summary

Students will be collecting data to calculate the flow rate of water leaking from a container. The primary focus will be on finding the uncertainty of the calculations, which is a quantity that measures the level of accuracy with which the data was collected.

Competencies

At the end of this less students will be able to:

1. describe the concept of uncertainty analysis
2. quantify an uncertainty for a given set of data
3. identify where errors may occur in collecting data
4. identify ways to limit error in data collection
5. organize data in tables and graphs
6. compute measures of central tendency
7. identify the slope and y-intercept of a linear relation in the table, graph, or equation
8. collect data measuring time and volume

Time

This lesson may take up to 3 hours to complete.

Materials

1. Graduated cylinders (1000 ml)
2. Two-Liter bottles with small holes
3. Stop watches
4. Handouts
5. Graph paper
6. Calculators (optional)
7. Spreadsheet software

Instructions

1. Give students the following situation: You are a mechanical engineer doing research on jet fuel for the Air Force. Our goal is to see if certain additives will increase the efficiency of the fuel. (Not for environmental reasons, but so the jet will perform at its best!) As you are running tests and collecting data, it is necessary for you to know exactly how much fuel is flowing to the engine. How can we calculate flow rate? How can we quantify our uncertainty from these measurements and calculations?
2. Ask students to throw out ideas on what needs to be done. (Record them all on a transparency so you can use them later.) Stress to the students that these ideas may be valuable when answering the questions after the activity.
3. Explain the two ways the class will be finding flow rate.
   a. Cylinder will be filled to a specific volume, time will be recorded
   b. Cylinder will be filled for a certain amount of time, volume will be recorded
4. Students will work on the handouts in groups of 3 to 4.

5. Groups will present their findings to the class. Each group should have a container with a slightly different size hole in it, so their data may be very different. Both graphs should be on one transparency. Explain that they will be assessed on the quality of the presentation, and on how well they describe what happened while the data was collected. (Be sure to ask the groups leading questions while they are working to prepare them for the presentations.)
6. Have students organize their organize their data in a spread sheet. At this point the groups should be working more independently on the spreadsheet. Share a few of these with the entire class. (Consider, if possible, having the students program the spreadsheet prior to collecting data either in this lesson or with the transfer activity.)

Evaluation/Assessment of Student's Competency

1. Each group will turn in a copy of their tables, graphs and questions to be graded.
2. Assess presentations based on how well they communicate their findings to the class… weather they are correct or not.
3. Peer evaluations of the presentations

Closure

Engineers will use flow meters to determine how much fuel is going into the engine. They calibrate the flow meters *using a similar process to what we did in this lesson*. The following equation is used to calculate **mass** flow,

$$M = P \bullet A \bullet C^* \bullet Cd \sqrt{\frac{g_c}{R \bullet T}}$$

where P = inlet pressure, A = throat area, C* = critical flow, Cd = discharge coefficient, gc = gravitational constant, R = gas constant, and T = inlet temperature. An equation with this many variables (fortunately a few are constants!) has much room for uncertainty. It is unlikely that all variables will be measured at the extreme high (or low) of their uncertainty range at the same time. (Perhaps this showed up in previous exercises.) The following formula can be used to narrow the uncertainty to a much more reasonable range.

$$U_M = \sqrt{\left(\frac{\partial M}{\partial P}U_P\right)^2 + \left(\frac{\partial M}{\partial A}U_A\right)^2 + \left(\frac{\partial M}{\partial C^*}U_{C^*}\right)^2 + \left(\frac{\partial M}{\partial Cd}U_{Cd}\right)^2 + \left(\frac{\partial M}{\partial g_c}U_{g_c}\right)^2 + \left(\frac{\partial M}{\partial R}U_R\right)^2 + \left(\frac{\partial M}{\partial T}U_T\right)^2}$$

This should look familiar to students that have taken statistics because of standard deviation, and most high school students should recognize this as being an extension of the distance formula. The U's with subscripts are the uncertainties associated with the given variable, and of course the other parts are the partial derivatives with respect to that variable. Though high school students typically know nothing about derivatives, you may simply explain that it is a way to measure the rate of change… how steep the graph is at a certain point. It's always fun to tell the story about Newton and Leibniz arguing over who really first came up with the ideas of calculus. Newton, falling apples, gravity, and on to the Transfer Activity!! (I think there may even be a School House Rock with Newton in it that you can show at this point☺)

Flow Rate
Volume Constant

1) Collect data and complete the table.

Volume (ml)	Time (sec)	Average Time (sec)	Rate (ml/sec)	Dist from Ave Rate	% from Ave Rate
100					
100					
100					
200					
200					
200					
300					
300					
300					
400					
400					
400					
500					
500					
500					
600					
600					
600					
700					
700					
700					
800					
800					
800					
900					
900					
900					
1000					
1000					
1000					
			Ave Rate =	Range =	Range =
Uncertainty =			Uncertainty % =		

2) Make a graph relating time (sec) and volume (ml). Use the Average Time column. Connect the dots as best as you can with at straight line. (What should the y-intercept of the line be?)

Flow Rate
Time Constant

3) Collect data and complete the table.

Time (sec)	**Volume** (ml)	Average Vol (ml)	Rate (ml/sec)	Dist from Ave Rate	% from Ave Rate
10					
10					
10					
20					
20					
20					
30					
30					
30					
40					
40					
40					
50					
50					
50					
60					
60					
60					
70					
70					
70					
80					
80					
80					
90					
90					
90					
100					
100					
100					
			Ave Rate =	Range =	Range =
Uncertainty =			Uncertainty % =		

4) Make a graph relating time (sec) and volume (ml). Use the Average Vol column. Connect the dots as best as you can with at straight line. (What should the y-intercept of the line be?)

Flow Rate
Questions

1) Should the rate at each time/volume average be the same? Why or why not?

2) What does the graph of a relation look like if the rate is constant? Why?

3) What can we observe about the uncertainty by looking at our graphs? Explain.

4) Compare the rates and uncertainties in the tables and graphs for both methods of collecting data.
   a) Which method, if either, do you feel was more reliable for checking the flow rate? Why?

   b) What other factors may have contributed to our uncertainty?

   c) How could we have improved the accuracy of our data collection? Explain.

Transfer Activity: Falling Objects

Summary

Students will be collecting data to calculate the flow rate of water leaking from a container. The primary focus will be on finding the uncertainty of the calculations, which is a quantity that measures the level of accuracy with which the data was collected.

The idea is for the groups to come up with the tables and methodology *on their own*. However, you may want to design some handouts to guide them if needed.

Competencies

At the end of this less students will be able to:
1. Collect data involving distance and time
2. Approximate a simple quadratic equation given a table of data
3. Find the equation that gives distance as a function of time for a falling object with no initial velocity.

Time

This lesson may take 5 hours not including the presentations.

Materials

1. Falling Objects (balls)
2. Stop watches
3. Graph paper
4. Handout
5. Spreadsheet and presentation software
6. Calculators

Instructions

1. Use the handout to brainstorm with the class on how to collect the data
2. Explain the expectations
3. Give the groups access to a high spot, such as bleachers, from which to conduct experiments. Monitor progress and give helpful suggestions when needed. Once the data is collected, students will either need to use algebraic techniques to approximate a quadratic equation or use the quadratic regression function on a graphing calculator.
4. Have students organize their data in a spreadsheet and use some type of presentation software to share what they did with the class.
5. Provide time for questions during the presentations.
6. Reveal the correct solution, $d = 1/2at^2$, for d is in feet, t is in seconds, and a is approximately $32ft/s^2$. Depending on the level of the class, explain the equation.

Evaluation/Assessment of Student's Competency

1. Students will be assessed individually on how they worked as a group. (Use peer evaluations)
2. Groups will be graded on the quality and accuracy of their experiments. (Does their uncertainty measurement reflect how far off their equation is?)
3. Groups will be graded on their ability to explain their findings to the class. (Again use peer evaluations in addition to teacher assessment.)

Closure

Conclude by showing students the three graphs of acceleration, velocity, and distance in relation to time. Acceleration is constant (horizontal), so velocity is linear (increasing), and distance is then quadratic. All begin at zero. The area under the acceleration graph from zero to time t is the final velocity at time t (integration from 0 to t). $v = \int_0^t a\,dt = at$. The area under the velocity graph from zero to time t is the final distance at time t (integration from 0 to t). $d = \int_0^t v\,dt = \int_o^t at\,dt = \frac{1}{2}at^2$. Simply build Riemann sums to illustrate why we are interested in the area under the curves for students with no calculus experience. Finally, graph their equations on a graph with the actual equation and discuss their uncertainty values.

Falling Objects

Transfer Activity

We know that ALL objects on Earth fall at the same constant acceleration, about 32.2ft/sec^2 or 9.81m/sec^2. This fact allows us to determine the distance an object has fallen given the time. Our goal is to find an equation that gives distance as a function of time for falling objects. The falling objects that we will be studying have no initial velocity.

- What kind of relation do you think this is? Why? (linear, quadratic, cubic, exponential, logarithmic, etc…)

- What data can we collect that will represent this relation?

- What should our procedures be? How can we design procedures that will minimize uncertainty?

- How will we quantify our uncertainty?

Instructions:
Working in groups, you will collect data that may lead you to an equation that gives distance as a function of time for fall objects. You will need to construct tables and graphs. In addition to handing in a beautifully written report explaining your procedure and data, your group will present your findings to the class. I'm sure you will find the spreadsheet software and presentation software to be very helpful. It is your job to convince us that your equation is correct… or at least pretty darn close… even if it's dead wrong! Therefore you will need to include some type of uncertainty analysis with your report.